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Beauty is not all there is to Aesthetics in Mathematics†

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ABSTRACT

Aesthetics in philosophy of mathematics is too narrowly construed. Beauty is not the only feature in mathematics that is arguably aesthetic. While not the highest aesthetic value, being interesting is a *sine qua non* for publishability. Of the many ways to be interesting, being explanatory has recently been discussed. The motivational power of what is interesting is important for both directing research and stimulating education. The scientific satisfaction of curiosity and the artistic desire for beautiful results are complementary but both aesthetic.

1. INTRODUCTION

I draw attention to the discrepancy in philosophy of mathematics between the two main uses of terms involving 'aesthetics' of which I am aware. On the one hand, it is a commonplace to admit or claim that 'aesthetic considerations' influence choices in mathematical practice not only in pure mathematics but also in applied mathematics, where exclusively utilitarian considerations might be expected. On the other hand, discussions of aesthetics within philosophy of mathematics are concerned nearly exclusively with discussions of beauty.

In looking at aesthetic literature in philosophy of mathematics (*e.g.*, [Plotnitsky, 1998]) in order to confirm that it mostly treats just beauty — however that is viewed — the closest that I have come to anyone's looking away from beauty is the essay by Nathalie Sinclair [2006] in the aesthetics volume that she co-edited [Sinclair *et al.*, 2006]. In their introductory chapter, the editors slip into writing of 'the aesthetic feeling' [Sinclair and Pimm, 2006, p. 12] as though

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there were only one. That does not seem to be Sinclair's considered view, as she writes of what is 'beautiful and interesting' [2006, p. 92] and acknowledges three different characteristics of 'the aesthetic', the *motivational* that attracts to what is not yet done and stimulates to do it, the *generative* that guides mathematical moves that are not deductive, and the *evaluative* that is the second-order appreciation of what has been done [2006, p. 89].

Having mentioned beauty, I should point out that I accept Rom Harré's view [1958] that judgements like that of beauty are 'second-order' (his term) without necessarily wishing to accept his downgrading them to 'quasi-aesthetic' (also his term). What I think Harré means by 'second-order' is that, in order to view a proof, say, as elegant, his favorite such feature, one needs to know it first and to appreciate that its simplicity has been artfully achieved even if one does not know a clumsier proof. This seems to be Sinclair's evaluation, which she also calls 'second-order'.

The above discrepancy is an example of the tunnel vision to which all scholarship is prone.[1] Once things are published that confine aesthetics of mathematics to beauty in mathematics, there is a tendency — has been a tendency — to maintain that narrowness. This tendency is well established in English [Todd, 2008].[2] The resolution of the discrepancy is obvious; aesthetics in mathematics needs to consider more than just what is beautiful. Those discussing a number of other features can admit that those features are aesthetic. The special issue of *Philosophia Mathematica* (**23** (2015), No. 2) on mathematical depth has no acknowledgement that its topic may sometimes be an aesthetic one, no indication of such a context, something one might expect even if it were argued about.

A reason why this is not done, which is a bad reason, is the low status of aesthetics in general within philosophy. This is bemoaned even by its enthusiasts [Devereaux, 1998].[3] Mary Devereaux points out that 'philosophers widely regard aesthetics as a marginal field.' She continues,

[1] An example of this must come from another topic. In 1898, H.B. Swete published an important commentary [1898] on the Gospel according to Mark, in which he accepted what evidence there was that the book was written in a particular place for readers *in that place*. For a century, the assumption that each of the four Christian gospels was written for a separate community increasingly dominated scholarship, leading to creative and sometimes fanciful constructions of these supposed separate communities, about which nothing was known. This scholarly consensus was first challenged by Bauckham [1998], on which the above sketch depends. Contradictorily, during the same century there arose the even more dominant view that two of the Gospels depend on that according to Mark, which had obviously circulated to wherever they were written, and the famous source Q, which was also accessible to both writers. I return to this in note 10.

[2] Most aesthetic literature on mathematics does not refer to anything before [Harré, 1958] except the book by Hardy [1967]. The situation is somewhat different in French; see, *e.g.*, [Sinclair, 2011], which cites [Hermite, 1905] (negative), [Poincaré, 1908] (heuristic), [Hadamard, 1945] (psychology), and [Le Lionnais, 1948] (taxonomy).

[3] I owe reference to Devereaux to Robert Kraut [2007].

> Aesthetics is marginal not only in the relatively benign sense that it lies at the edge, or border, of the discipline, but also in the additional, more troubling, sense that it is deemed philosophically unimportant. In this respect, aesthetics contrasts with areas like the philosophy of mathematics, a field which, while marginal in the first sense, is widely regarded as philosophically important.

To be at the margin of the margin is to risk falling off the edge. (The cure for that is to be edgier.) Whatever may be the case within philosophy in general, if one is to consider mathematical practice, which is performed by humans with values, their mathematical choices must come into play, and those choices are based on aesthetic considerations, among others. Aesthetics in mathematical practice is not marginal in either sense.

My argument is for two theses. 'Intresting' is an aesthetic feature seen in all published mathematics. This may be interesting, but I do not see it as important in itself. It implies my second thesis, that beauty is not all there is to aesthetics in mathematics any more than beauty is all there is to aesthetics outside of mathematics. This I do not see as interesting, but the default contrary view is importantly wrong. (Not to mention the further limitation to proofs.)

2. BEING INTERESTING IS AN AESTHETIC VALUE

In order to claim that attribution of aesthetic considerations unrelated to beauty is not just eccentric because aesthetic considerations *have* to relate to beauty, I offer some second-hand historical justification. The book, *The Future of Aesthetics* by Francis Sparshott [1998] actually says enough about the past of aesthetics for present purposes. The term 'aesthetik', he says, was coined by Alexander Gottlieb Baumgarten [1735] to cover the non-logical side of philosophical considerations, matters of more and less rather than yes and no. This is plainly the sense in which what are called still 'aesthetic considerations' influence mathematical practice. How it has survived the narrowing of aesthetics to beauty and the arts already in the eighteenth century I do not know. Some word has to cover non-logical matters of degree, and 'aesthetics' has remained available. 'Valid' of a proof is an evaluative term indicating a position on a Boolean scale. Aesthetic evaluations are those that are not logical but have to do with a scale of merit. 'Short' of a proof is descriptive and not logical, but it is not in itself an aesthetic judgement. To say, however, that one proof is shorter than another, *ceteris paribus*, could be an aesthetic evaluation, if only a minor one. How minor would depend on the lengths compared. Almost any conceptual proof of the four-colour theorem would be shorter and aesthetically preferable, while being preferable in other respects as well.

I claim that, as well as 'beautiful' and sometimes 'deep', 'interesting' is an important aesthetic category — indeed that 'interesting' is a *sine qua non* of publishable mathematical research. The main basis for this claim is my ten years of experience as managing editor of a mathematics journal. For the whole of that time, the criteria that I asked referees to use in recommending acceptance of a manuscript were whether it was original, correct, and interesting. One does not want to publish what has already been published or what is wrong or what is new and correct but of no interest. Arithmetic alone supplies an infinite sequence of such results, since there are always larger and larger number pairs that have not been added. If this were research, M.Sc. theses could do additions and Ph.D. theses subtractions. These sums and differences must be rejected on the ground of lack of interest, since they cannot be rejected for being either wrong or well-known. While these particular results lack interest because the method of *producing* them is well-known, I say without fear of contradiction that there are other ways to lack interest. Writers create documents of some originality that are mathematical but lack mathematical interest; sometimes they submit them to a philosophy journal.

Perhaps being interesting, in spite of being a matter of degree, is merely an epistemic condition; that would get it out of the aesthetic box. It is partly but not just epistemic, because the uninteresting sums do tell their reader something that the reader did not already know, and the mathematical results that their authors think are of philosophical though not mathematical interest do not follow well-trodden paths. They do tell a reader something that the reader definitely does not know but probably does not want to know. Not wanting to know something is, it seems to me, a negative aesthetic judgement.[4] One can hardly deny an epistemic component to mathematical interest, but different mathematical results *feel* different. Learning mathematics does not necessarily make one feel interested, as generations of school leavers attest. Being interesting is not just an epistemic condition, but is it associated with other aesthetic judgements? As soon as I formulated this question in the spring of 2015, evidence came unbidden.

In the then current issue of *Philosophia Mathematica*, the paper on mathematical beauty [Inglis and Aberdein, 2015] has the second sentence, 'Mathematicians talk of "beautiful", "deep", "insightful", and "interesting" proofs, and award each other prizes on the basis of these assessments.' Despite the fact that being interested is very much a matter of how one feels, 'interesting' does not occur again in the paper. In particular, it is not one of the eighty adjectives compared with 'beautiful'. I do not disagree with the authors that interest seems orthogonal to beauty. But they clearly regard it as aesthetic anyway.

In the then current issue of *The Mathematical Intelligencer*, being interesting is singled out in the cooking column [Henle, 2015] as an aesthetic quality common to mathematics and wine. In both cases one needs some level of sophistication to find interest rather than just learning something or quenching thirst. Having made this point about wine, mathematician Jim Henle continues,

[4] It is perhaps clearer that the decision that a *question* is uninteresting is non-epistemic since in the question there is no knowledge to call forth an epistemic judgement.

> Mathematics is much the same. It's more than useful; it's engaging. The fact that two plus two is four satisfies a primitive need, but a complex mathematical structure holds our interest. Mathematical ideas are enigmatic and charming. They yield treasures and they keep secrets. Mathematical structures appear different in different contexts. Local changes force global transformations. Mathematics entertains us and we treasure its mysteries.

The same, but also different. One difference is that interest for mathematics is essential, beauty an option; for wine, a pleasing flavour is essential with interest an option. The distinguished aesthetician Frank Sibley devoted a long time to a substantial essay [2002a], which he died before publishing, on the aesthetic value of tastes and smells. Jim Henle did not make up either side of his analogy.

There are two verbal matters to do with 'interest' that need to be mentioned. Aesthetic judgements, like other judgements that are meant to have objectivity or intersubjectivity, are supposed to be disinterested. How then can a judgement of interest be disinterested? Because the sense of 'interest' that disinterest avoids means dependent on the thought of or 'desire for the use or possession of their objects' ([Cooper, 1711] as quoted in the *Stanford Encyclopedia of Philosophy* article '18th century British aesthetics' of 2014). Accordingly, 'disinterested interest' is no oxymoron, only a limitation. Disinterest will return.

The other verbal matter is the complementarity of ways, independent of those of the previous paragraph, in which we use 'interest'. As I have often told students of a compulsory course that they would not have chosen, one can be passively the prisoner of what interests one willy nilly, as everyone knows, or one can deliberately *take an interest* in something that one decides to pay attention to, like a course that one is taking, for example. The latter is an act of will that is carried out by reading attentively and perhaps doing exercises, but surprisingly often (always in my experience) it leads to becoming reactively interested by the material. An example is my recent experience of the *Spherics* of Theodosios [Ver Eecke, 1959]. This second-century BCE treatise on circles on a sphere, although it held its place in the quadrivium as long as that lengthy tradition lasted, has been little thought of since. It did not reactively interest Thomas Heath in particular [1921, vol. 2, pp. 248–252], and he did not take an active interest in it. A few years ago, I took sufficient interest in it to translate it from Greek.[5] Definitely an act of will; my Greek is not good enough for any document to entice me to study it. But when I became familiar with it I found that it had mathematical interest enough to want to pass it on. As a result, I have written something I call an appreciation of the first of its three books. While that is not yet published, two referees have agreed (I am told) that it is of sufficient interest to be worth publishing. The more one knows about something valuable and mathematical, the more interest one finds in it. Mathematics that

[5] It had already been translated from a Latin translation in the eighteenth century, but I did not know that.

is not of sufficiently *general* interest is published in specialized journals where readers know enough about the subject to be reactively interested or to take an active interest.

The sense of 'interest' at issue can perhaps be conveyed more clearly in terms of attention. It is an important feature of advertising and journalism to attract attention, to interest a person in the reactive sense. Good journalism and much other writing attempts to hold attention, to develop the reader's interest after initially grabbing it. This is our topic here, although since, being reactive, it must be begun; a bit of grabbing is part of the package. 'It is a truth universally acknowledged, that a single man in possession of a good fortune must be in want of a wife.' Interest in anything whatever in the active sense is always possible; one can pay attention to whatever one wants to pay attention to. And finally, the kind of interest to which attention is irrelevant is the kind referred to in the phrase 'one's own best interests'. Attention may advance those interests, but it does not help to define them. This is the interest avoided in disinterest.

In these terms, it is reactive interest that indicates the aesthetic value. Disinterested aesthetic contemplation is non-trivial to describe. Sibley quotes [2002a, p. 230] with approval this description.

> Thus we may define an interest in an object X for its own sake as a desire to go on hearing, looking at, or in some other way having experience of X, where there is no reason for this desire in terms of any other desire or appetite that the experience of X may fulfil, and where the desire arises out of, and is accompanied by, the thought of X... I shall respond to the question 'Why are you interested in X?' ... with a description of X. [Scruton, 1974, p. 148]

I believe that the sense of 'interested in' in the question is 'paying attention to'. Scruton is not arguing, with me, that being interesting is itself an aesthetic value. When giving a description of X to explain why something is of aesthetic value, one frequently mentions non-aesthetic characteristics. This is true also of why something mathematical is interesting.[6]

Scruton's description illustrates that aesthetics cannot have firm boundaries. This is a fact (no necessary or sufficient conditions) emphasized by Sibley [1959], 'Accordingly, when a word or expression is such that taste or perceptiveness is required in order to apply it, I shall call it an aesthetic term or expression, and I shall, correspondingly, speak of aesthetic concepts or taste concepts.' I am claiming that the aesthetic has been drawn much too narrowly in discussion

[6] Sibley wrote a whole essay [1965] on the mysterious relation in the arts between aesthetic and non-aesthetic properties, in particular that, as Scruton says, one's explanation of aesthetic value is descriptive, and, as Sibley says, that description involves mainly non-aesthetic properties, which somehow add up to something aesthetic. A mathematical example of this is that, in explaining why a proof is interesting, one might invoke the so-called purity of its method

of mathematics. What is interesting in mathematics and *how* interesting (*i.e.*, more or less rather than in what way) are very much matters of trained taste.

'Interesting' is not the same thing as the more objective property 'important', which is not so subject to disagreement. The taste of wine is not in any absolute sense important, however interesting it may be, but it is of enormous commercial importance. Likewise, a piece of mathematics can be of importance either for application outside mathematics or for mathematical use, but it can be interesting without having either of those non-aesthetic values. Unfortunately, importance is paid more attention than interest in mathematical education. Since a most important aspect of teaching is the engagement of students' interest, this is a mistake. The good effects of Martin Gardner on young readers, now alas all grown up, are widely reported by them. Gardner, in his column in the *Scientific American* from 1956 to 1981, chiefly revealed the interest inherent in the topics he chose to expound. No one suggests that he manufactured that interest. He had a nearly unique ability to find and expose it and in consequence interested thousands of persons that became mathematicians and many more others. Education needs elements of this skill, as Gardner himself maintained [Mulcahy and Richards, 2014]. Discussion on what is interesting (and how) is probably a necessary preliminary. This has not been being done. An exception is [Wells, 2015, Ch. 3–6].

As I am not concerned here with beauty, I merely remark that mathematical interest is not infected with the difficulties that feminists can and do find in male-gendered considerations of beauty since Plato's *Symposium* [Sparshott, 1998, p. 15].

I have set out the fact that being interesting is regarded by some as an aesthetic value; it seems to me that such evidence is more important than argument. But as it happens there is an argument available, for what it is worth in such a context. It is rightly said that aesthetic considerations weigh with mathematical researchers when doing research and not just afterwards. I claim that those considerations, while they may sometimes have to do with beauty, more frequently have to do with judgements of what will be interesting. That is to say that research is driven more by curiosity-based first-order judgements than second-order judgements that can only be made of results. It is incoherent to claim that in searching for one knows not what (otherwise it would not be research) one is strongly influenced by the appearance of what is to be found. One could easily be influenced by what one *hopes* to find, and no doubt often is, but that is as much about its interest as its potential beauty. Conjectures are important, but they are not starting points. One way that hopes can work is to motivate adjusting premises to allow preferable results or proofs.[7]

It would be courteous to remark that Sinclair's 'motivational' and 'generative' characteristics of the aesthetic are close to 'being interesting', since being interesting applies to what one has not yet done and I think also to what one has not yet read. Even reading mathematics requires some motivation. Surely

[7] [Rota, 1997, p. 178], quoted in [Cellucci, 2014].

one cannot appreciate as beautiful what one has not yet done (and may not do) or what one has not yet read? In both cases one can be interested and often is.

I have no desire to put down beauty, only to elevate interest from invisibility to its place of importance. There are different ways to be interesting, but this is not the place for a catalogue. One way currently discussed is explanatoriness [Hafner and Mancosu, 2005; Tappenden, 2008; Baker, forthcoming]. It is regarded as a value of proofs almost universally, although like beauty [Rota, 1997], it can be put down [Zelcer, 2013]. Proofs can be explanatory (or not [Resnik and Kushner, 1987][8]) of what they prove, but also results can explain other results like the interval of convergence of the series for $1/(1+x^2)$. Much philosophical discussion of explanation extends from mathematics to physical *explananda* [Baker, 2005], but that need not concern us here. Within mathematics various ways to be explanatory are identified in [Hafner and Mancosu, 2005, Section 3], where distinct ways of being explanatory are put forward, the evidence coming down to the fact that mathematicians find them to be explanatory to some degree. All of their analysis is about just one way of being interesting. That there is so much room for differences of opinion is evidence that being explanatory and interesting more generally are aesthetic qualities.

It must be admitted that, as the use of 'interesting' in fiction shows, it is not the highest aesthetic value there. Crime fiction, fantasy, romances, and science fiction are often page-turners without being held to be of great literary value. It is unlikely that it is the highest praise for mathematics either. Both beauty and depth are more highly valued and less time-dependent [Wells, 1988, 1990]; all I am saying is that being interesting is necessary. The booklet *A Manual for Authors of Mathematical Papers* [AMS, 1990] warns not to try to publish detail that it is good to work out 'since it is likely to be long and un-interesting' (quoted by Sinclair [2011]).

It is no part of my claim for interest that only mathematics is interesting. I have no idea even whether it is uniquely important in mathematics. In history, literary criticism, or any other discipline, what is written has also to be interesting to be publishable. While being written interestingly is a positive feature no doubt, the bad reputation of academic writing in general suggests that the interest needed in other subjects is in the subject matter for reasons to do with that subject matter. A history essay is of more interest as the events described are of more importance. Literary criticism is of more interest as the literature discussed is better. Economics is of more interest as the phenomena explained are more widespread or important in some other way. Mathematics can be of interest for this sort of reason too. There is limited interest for its own sake in the unsolvability in integers of Pythagoras-like relations for higher powers

[8] The negativity of Resnik and Kushner seems to be based on their notion that explanatoriness is a matter of yes and no rather than of degree, which it is as an aesthetic feature. They quote Davis and Hersh [1982, p. 299] before explanatoriness became a common concern as writing that the prime-factorization proof of the irrationality of $\sqrt{2}$ 'exhibits a higher degree of aesthetic delight' than the Pythagorean proof because it 'seems to reveal the heart of the matter'. What they clearly regarded as a matter of degree, since they use the word, I venture would later have been termed 'explanatoriness'.

than two, but the proof of Fermat's famous conjecture was of great interest because centuries of effort had rendered it important beyond its raw material. As in many other examples of important mathematical accomplishments, active interest was taken on account of the importance. I do not mean to suggest that this active kind of interest is aesthetic, but that the reactive sort is. It is also the more frequent motivation — especially in pure mathematics.

3. CONCLUSION

I conclude with a pair of contrasting analogies. Much mathematical effort is more like landscape gardening than like picture drawing. I take picture drawing to begin with a blank sheet on which the artist represents something imagined or seen, a chief aim being to create something of value. The artist is free, because the page is blank, in the choice of what is to be represented, which need not be something seen, and in how it is represented. Mathematical creation is not so free, hence the contrasting analogy of the landscape gardener, who needs a good grasp of the topography before getting down to creating something beautiful, which needs to be based on that topography. When H.S.M. Coxeter handed out copies of the preliminary edition of his book *Projective Geometry* [1974] to his undergraduate students in 1963, the preface included a sentence to the effect that the only mention of cross ratio in the book was in that sentence.[9] He drew our attention to this, regarding it as aesthetically pleasing to avoid all use of cross ratio in his landscaped development of elementary projective geometry. He knew the terrain well and was able to accomplish this aim because of his mastery of it. But Desargues had been dead for three hundred years; much projective geometry had been done with the aim of understanding the topography, curiosity-driven research that had the aim of finding interesting projective properties, initially in Euclidean space and certainly not avoiding cross ratio. It is in the aesthetic space I have been writing about that the complementary scientific pursuit of what is interesting and artistic pursuit of what is beautiful interact. A lot of mathematics does not get past mapping the topography. G.H. Hardy is often quoted as writing 'Beauty is the first test: there is no permanent place in the world for ugly mathematics' [1967, p. 85]. Being interesting is apparently test zero, because without being interesting it is not even ugly mathematical research. *Eventually* one wants everything to be beautiful and so to be permanent, but these things can take time. The problematic status of the parallel postulate was regarded as an aesthetic blemish on geometry for over two thousand years before it was eliminated by clarifying that there is more to geometry than the *Elements*.[10] *Ars longa; vita brevis.*

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[9] Disappointingly, the sentence was soon changed to, 'In particular, the only mention of *cross ratio* is in three exercises at the end of Section 12.3.'

[10] Just as there is more to aesthetics than beauty. This is a further example of what was discussed in note 1. It could have been noticed at any time after the writing of the first book of *Spherics* (almost certainly before Euclid, who uses results from what are now the second and third books) that the postulates other than the parallel postulate are satisfied by points and great circles on a sphere, but attention was reserved for the intended model.

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